Schweizer 1-26 Moment of Inertia Determination

By Frank Sanders

Introduction

The purpose for finding the moment of inertia of the Schweizer 1-26B sailplane is to implement the values for the impending research once the finished aircraft is flying. The intent of the flight research with the aircraft is to identify the stability and control derivatives of the aircraft. In order to find the derivatives we will use frequency response techniques to generate the transfer functions that relate control inputs and the aircraft's response. The frequency response techniques require that the aircraft be flown in such a way as to excite the frequency modes of the airframe. From the data collected from the frequency sweeps, we will use a program called CIFER (Comprehensive Identification from Frequency Response) to create the transfer functions that model the aircraft's dynamics in response to the control inputs. We can extract the stability and control derivatives from the transfer functions created in CIFER. Moment of inertia is critical because the moment coefficients are directly dependent on the moment of inertia. For example, Equation 1 is the expression of the pitching moment of the aircraft with respect to the deflection of the elevator, or the elevator power.

Equation 1:

$$M_{\delta_e} = \frac{\overline{q} S \overline{c} C_{M_{\delta_e}}}{I_y}$$

Moment of Inertia Determination

To start the process, I designed the frame in AutoCAD, since I had drawn a full scale model of the 1-26 in AutoCAD to determine the stabilizer and flight control areas for the performance calculations. The model allowed me to draw the frame to scale so that I could buy the appropriate amount of wood.

To determine the moment of inertia of the aircraft I consulted NACA TN No. 351 entitled "An Accurate Method of Measuring the Moments of Inertia of Airplanes." In this report, the theory and approach to determining the moments of inertia of an aircraft are presented and discussed. The method is essentially a superposition method, for one swings the aircraft and frame together, then swings the frame by itself, calculating MOI each time. Then one subtracts the MOI of the frame from the MOI of the frame and aircraft together, leaving just the MOI of the aircraft. The theoretical and experimental calculations can be found at the end of the explication. As simple as that explanation sounds, the actual process of swinging the airplane and collecting data was quite long and tiring.

To collect data, NACA suggests deflecting the aircraft and then allowing it to oscillate 50 times while having two people recording the period of oscillation with a stop watch. Also, the calculations require that the aircraft be swung at two different heights, high and low. So, a colleague of mine from SJSU had built an Arduino microcontroller based period of oscillation computer, using an optical sensor that senses a transition from light to dark as a counter, which computes the period of oscillation and then displays it on a small LCD screen. We used this computer and paper counter strips (black bars with a white strip in the middle) taped to the fuselage to count and compute the period of oscillation.

With each axis, we swung the airplane five times per frame height to ensure replication of results. With an average period of four seconds and 50 oscillations per run, we had to wait about three and a half minutes per swing. Once we swung the airplane five times, we set it down and unbolted the wing saddles and moved them accordingly. We needed to collect data for both the x-axis and y-axis (about the wing and fuselage respectively), so we transitioned the hinge 90 degrees so it would roll instead of pitch. By the end of the day, we had swung the airplane about 30 times, and we were well exhausted with the process.

Future Work

Once the turbine addition work is complete, we'll swing the aircraft again with four different configurations: empty with turbines retracted, empty with turbines extended, full fuel turbines retracted, full fuel turbines extended. Since the data was so consistent with the empty aircraft, we'll only need to swing the aircraft four times per configuration. Also, hopefully, we'll be able to swing the airplane about the z-axis to determine the yaw axis stability and control derivatives.

MOI Theoretical Predictions

Rectangular Block:

$$I_x = \frac{1}{12} m (y^2 + z^2)$$

$$I_y = \frac{1}{12} m (x^2 + z^2)$$

Wing:

$$mass = \frac{247\ lb}{32.2\ ^{ft}/_{sec^2}} = 7.6708\ slugs$$

y= 40ft

x= 5ft

z= 0.83ft

$$I_x = \frac{1}{12} 7.6708 (40^2 + 0.83^2) = 1023.2\ slug * ft^2$$

$$I_y = \frac{1}{12} 7.6708 (5^2 + 0.83^2) = 16.42\ slug * ft^2$$

Fuselage:

$$mass = \frac{247\ lb}{32.2\ ^{ft}/_{sec^2}} = 4.3788\ slug$$

y= 21.5ft

x= 1.5ft

z= 2.5ft

$$I_x = \frac{1}{12} 4.3788 (1.5^2 + 2.5^2) = 3.102\ slug * ft^2$$

$$I_y = \frac{1}{12} 4.3788 (21.5^2 + 2.5^2) = 170.96\ slug * ft^2$$

$$I_y = 170.96 + 4.3788(3^2) = 210.62\ slug * ft^2$$

Horizontal Tail:

$$mass = \frac{15\ lb}{32.2\ ^{ft}/_{sec^2}} = 0.46584\ slug$$

y= 9.5ft

x= 4ft

z= 0.083ft

$$I_x = \frac{1}{12} 0.46584 (9.5^2 + 0.083^2) = 3.5\ slug * ft^2$$

Horizontal Tail Cont.

$$I_y = \frac{1}{12} 0.46584\,(4^2 + 0.083^2) = 0.62\ slug * ft^2$$
$$I_y = 0.62 + 0.46584(9.5^2) = 42.66\ slug * ft^2$$

$$I_{x,theoretical} = 1023.2 + 3.5 + 3.102 = \mathbf{1029.60\ slug * ft^2}$$
$$I_{y,theoretical} = 16.42 + 42.66 + 210.62 = \mathbf{296.7\ slug * ft^2}$$

MOI Calculation based on Experimental Data

$$I = \frac{W_1 * {T_1}^2 * L_1}{4\pi^2} - \frac{W_2 * {T_2}^2 * L_2}{4\pi^2} - \frac{W_3 * {L_3}^2}{g}$$

X-axis

Variable	**Short**	**Long**
W_1	961.5 lb	961.5 lb
W_2	318.5 lb	318.5 lb
W_3	643 lb	643 lb
L_1	5.259 ft	6.278 ft
L_2	3.126 ft	3.428 ft
L_3	6.316 ft	7.691 ft
T_1	3.9585 sec	3.9864 sec
T_2	3.2629 sec	3.3316 sec

Y-axis

Variable	**Short**	**Long**
W_1	961.5 lb	961.5 lb
W_2	318.5 lb	318.5 lb
W_3	643 lb	643 lb
L_1	5.354 ft	6.209 ft
L_2	2.997 ft	3.557 ft
L_3	6.522 ft	7.522 ft
T_1	3.1767 sec	3.3053 sec
T_2	2.9555 sec	3.0491 sec

Ix

$$I_{x,short} = \frac{961.5 * 3.9585^2 * 5.259}{4\pi^2} - \frac{318.5 * 3.2629^2 * 3.126}{4\pi^2} - \frac{643 * 6.316^2}{32.2}$$

$I_{x,short} = 941.93\ slug * ft^2$

$$I_{x,long} = \frac{961.5 * 3.9864^2 * 6.278}{4\pi^2} - \frac{318.5 * 3.3316^2 * 3.428}{4\pi^2} - \frac{643 * 7.691^2}{32.2}$$

$I_{x,long} = 944.52\ slug * ft^2$

$I_x = 943.23\ slug * ft^2$

Iy

$$I_{y,short} = \frac{961.5 * 3.1767^2 * 5.354}{4\pi^2} - \frac{318.5 * 2.9555^2 * 2.997}{4\pi^2} - \frac{643 * 6.522^2}{32.2}$$

$I_{y,short} = 255.28\ slug * ft^2$

$$I_{y,long} = \frac{961.5 * 3.3053^2 * 6.209}{4\pi^2} - \frac{318.5 * 3.0491^2 * 3.557}{4\pi^2} - \frac{643 * 7.522^2}{32.2}$$

$I_{y,long} = 254.64\ slug * ft^2$

$I_y = 254.96\ slug * ft^2$

Percent Error between Prediction and Experimental Values

$I_x = 9.2\%$

$I_y = 5.8\%$